2nd QPR

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~~3rd~~ QUARTERLY PROGRESS REPORT

Contract ~~No.~~ NSR-05-264-002

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# STUDY OF SHAPE AND INTERNAL STRUCTURE OF MOON, UTILIZING LUNAR ORBITER DATA

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LUNAR ORBITER PROGRAM OFFICE

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# LETTER OF TRANSMITTAL

Office of Grants and Research
National Aeronautics and Space Administration
Washington, D. C. 20546

Attention: NASA Technical Reports Officer, Code SC

Subject: 3rd Quarterly Progress Report
Contract No. NSR 05-264-002

Gentlemen:

Earth Science Research Corporation (ESR) is pleased to submit its 3rd quarterly progress report of a study of the shape and internal structure of the Moon utilizing data from the Lunar Orbiter Program under NASA Contract No. NSR 05-264-002.

It was found that the $J_{3,0}$ term determined from study of the orbit of Luna 10 has the same sign as the value which we would have predicted from the relationship between the continent-maria distribution and orientation of the principal moments of inertia Equations have been derived which relate the zonal harmonics of the gravity field to similar terms in a lunar continentality function and the parameters of pausible models of internal structure. A computer program has been written and tested which determines the higher order terms in a continentality function for the Moon. A review of previous studies of the shape of the Moon is continuing.

Very truly yours,

EARTH SCIENCE RESEARCH CORPORATION

Donald L. Lamar

Donald L. Lamar
Principal Investigator

## RELATIONSHIP BETWEEN CONTINENT-MARIA DISTRIBUTION AND GRAVITY FIELD

**Results:**

Analysis by Akim (1966) of the orbit of Luna 10 has produced the following values and maximum errors for eleven coefficients of the expansion of the lunar gravity field:

$$
\begin{aligned}
c_{20} &= (-0.206 \pm 0.022) \times 10^{-3} \\
c_{21} &= (\ 0.157 \pm 0.059) \times 10^{-4} \\
d_{21} &= (\ 0.361 \pm 0.358) \times 10^{-5} \\
c_{22} &= (\ 0.140 \pm 0.012) \times 10^{-4} \\
d_{22} &= (-0.139 \pm 0.145) \times 10^{-5} \\
c_{30} &= (-0.363 \pm 0.099) \times 10^{-4} \\
c_{31} &= (-0.568 \pm 0.026) \times 10^{-4} \\
d_{31} &= (-0.178 \pm 0.032) \times 10^{-4} \\
c_{32} &= (\ 0.118 \pm 0.047) \times 10^{-4} \\
d_{32} &= (-0.702 \pm 4.595) \times 10^{-6} \\
c_{40} &= (\ 0.333 \pm 0.270) \times 10^{-4}
\end{aligned}
$$

The values of $d_{22}$ and $d_{21}$ are close enough to zero to be consistent with the Moon's stable orientation in space and lack of noticable wobble. The $c_{21}$ term should also be close to zero if the axis of rotation is coincident with a principal moment of inertia. The value of $c_{30}$ is opposite is sign from the value of $1 \times 10^{-4}$ reported by Michael, Tolson and Gapcynski (1966) from study of the orbit of the first United States satellite of the Moon. According to William Michael (personal communication, 1967) the inclination of the orbit of this satellite was too low to permit a satisfactory determination of the Moon's gravity field.

The sign of $c_{30}$ reported by Akim is the same as we would have predicted on the basis of denser material beneath the Maria to explain the relationship between the distribution of continents and the orientation of the Moon's principal moments of inertia and the absence of Maria on the Moon's far side (Earth Science Research Corp., 1967). That is, Akim's value of $c_{30}$ is consistent with our earlier suggestion (Lamar and McGann, 1966) that the material beneath the maria is denser than that lying beneath the continents.

In order to estimate simple mass distributions to produce the values of the higher order terms in the gravity field it is necessary to derive equations analogous to those presented in an earlier paper for the second order terms (Lamar and McGann, 1966). From Earth Science Research Corp. (1966) the effect on the gravity field of a surface density ($\sigma_n$) on a sphere of radius r is

$$J_n = - \frac{4\pi \ \sigma_n r^{n+2}}{(2n+1)\ MR^n}$$

where R is the Moon's radius and M is the Moon's mass; $J_n = -c_{n,0}$. In integral form:

$$J_n = - \frac{3\, a_n^0}{(2n+1)\ \rho_a \ R^{n+3}} \int_0^{R+H} r^{n+2}\, \Delta\rho \ (r)\, dr$$

where $\rho_a$ is the average density of the Moon, $a_n^0$ is a zonal term in a lunar continentality function, R is the radius of the Moon, $\Delta\rho$ (r) is the density contrast between continents and maria, and H is the average difference in elevation between continents and maria.

For the rigid lumpy model we assume a linear decrease in the density contrast with depth, or $\Delta\rho(r) = \Delta\rho(R)(r/R)$ where $\Delta\rho$ (R) is the density contrast at the surface; thus by integration:

$$\Delta\rho(R) = - \left[ \frac{(2n+1)\ \rho_a\ J_n^0}{3\, a_n^0} + \frac{H\ \rho_a}{R} \right] (n+4)$$

we assume that the density of material near the surface is equal to the average density of the Moon.

For the extreme case of a rigid, lumpy model we assume that the maria are directly underlain by a layer of nickel-iron. The thickness (d) of the layer is found to be

$$d = \frac{\rho_a \ P}{(\rho_i - \rho_a)} \left[ H + \frac{(2n+1)}{3 a_n^0} J_n^0 \ R \right]$$

where $\rho_i$ is the density of nickel iron.

In the case where topographic irregularities are assumed to be compensated by variations in the thickness of a floating crust we assume zero thickness for the crust beneath the maria, compensation at a depth D beneath the continents and a density contrast between continents and maria which decreases linearily with depth; by integration

$$\Delta\rho(R) = \frac{R(n+4)}{(R-D)^{n+4}} \left[ \frac{(2n+1)\,\rho_a \, R^{n+3} J_n^0}{3 a_n^0} \right] + \left( \frac{\rho_c}{n+3} \right) \left[ (R-H)^{n+3} - R^{n+3} \right]$$

$$+ \left( \frac{\rho_c - \rho_{mm}}{n+3} \right) \left[ R^{n+3} - (R-D)^{n+3} \right]$$

where $\rho_c$ is the density of crustal material and $\rho_{mm}$ is the density of material directly beneath the maria. We assume a condition of zero stress at the Moon's center; from Lamar and McGann (1966)

$$\Delta\rho(R) = \frac{3R}{(R-D)^3} \left\{ \frac{\rho_c}{2} \left[ (R+H)^2 - R^2 \right] \right.$$

$$\left. - \frac{(\rho_{mm} - \rho_c)}{2} \left[ R^2 - (R-D)^2 \right] \right\}$$

Note that for the $a_2^0$ term and the relations:

$$J_2^0 = \frac{2C - (A + B)}{2MR^2}, \qquad \beta = \frac{C - A}{B} \simeq \frac{C - A}{C},$$

$$\gamma = \frac{B - A}{C}, \qquad C \approx \frac{2}{5} MR^2$$

where C, B, and A are the principal moments of inertia of the Moon, the above equations reduce to those presented in Lamar and McGann (1966).

Work in Progress:

A computer program has been written which will determine the terms for all values of n and m in a lunar continentality function with the Moon divided into $20^{o}$ squares. The program is being modified to reduce the core storage required so that smaller squares can be used. The above equations will also be incorporated into the program so that we can relate the terms in the gravity field to density variations within the Moon for a number of plausible models of internal structure.

## REFERENCES


Akim, E. L. (1966) Determination of Gravitational Field of the Moon from the Motion of the Artificial Lunar Satellite Luna 10; translated by J. C. Noyes from: Doklady of the Academy of Sciences of the USSR, Vol. 170, No. 4, pp. 799-802.

Earth Science Research Corp. (1966) 1st Quarterly Progress Report, Contract No. NSR 05-264-002.

Earth Science Research Corp. (1967) 2nd Quarterly Progress Report, Contract No. NSR 05-264-002.

Lamar, D. L. and Jeannine McGann (1966). "Shape and Internal Structure of the Moon," Icarus, Vol. 5, p. 10-23.

Michael, W. H., R. H. Telson, and J. P. Gapcynski (1966). "Lunar Orbiter: Tracking Data Indicate Properties of Moon's Gravitational Field," Science, Vol. 153, p. 1102-1103.